to the inverse. Also, the contrapositive is logically equivalent to the conditional.

Logical equivalence means that two statements (composed of propositions) share the same truth table.

Homework: (Due Mon, Jan 19th)

PP. 33-35 # 29, 31, 33, 40, 42, 44, 58, 64, 67, 75-77, 83, 84, 88, 98, 99, 100.

§ 1C Sets and Venn Diagrams

Definition: Set - a collection of objects with some commonality.

Set notation: One way to represent sets is using brace notation and ellipsis.

Ex: The set of all magnum letters in the English alphabet

$\{A, B, C, \ldots, Y, Z\}$

Ex: The set of counting numbers

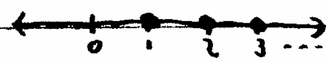
$\{1, 2, 3, \ldots\}$

The first example uses ellipsis to represent the letters D through X. The second example uses ellipsis to represent all integers larger than 3.

There are 6 sets of numbers that are vital to our studies. They are:

NATURAL NUMBERS (Counting Numbers)

Set Notation: $\{1, 2, 3, \ldots\}$

Number Line: 

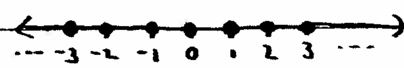
WHOLE NUMBERS

Set Notation: $\{0, 1, 2, 3, \ldots\}$

Number Line: 

INTEGERS

Set Notation: $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Number Line: 

Notice that each number line INCLUDES the ellipsis from the set notation. This is vital. For instance:

Ex: $\{0, 1, 2, 3, \ldots\}$ is NOT the same set as $\{0, 1, 2, 3\}$. The former includes all integers from 0 to infinity. The latter only includes the numbers 0, 1, 2, and 3. Likewise,

is NOT the same

as 

RATIONAL NUMBERS

The rational numbers are given as a definition. We will NOT use set notation for the entire set of rational numbers.

Definition: A rational number is any number that can be written as $\frac{r}{s}$, where r and s are integers and $s \neq 0$.

Ex: $\frac{1}{2}$ is rational since 1 and 2 are both integers.

Ex: $-\frac{23}{4}$ is rational since 23 and 4 are both integers. The negative has no influence. Ignore it.

Ex: $\frac{.5}{2}$ is rational. 2 is in the set of integers. .5 is <u>not</u> an integer, but we can use fraction manipulations to make it an integer. Multiply by 1:

$$\frac{.5}{2} = \frac{.5}{2} \cdot 1 = \frac{.5}{2} \cdot \frac{10}{10} = \frac{5}{20}$$

Since 20 and 5 are integers. It follows $\frac{.5}{2} = \frac{5}{20}$ is rational.

Ex: $-\frac{3}{0}$ is NOT rational since, by the given definition of rational number we cannot divide by 0.

Ex: Notice that the Natural Numbers, Whole Numbers, and integers are all Rational Numbers.

Decimal representation of Rational Numbers:

All Rational Numbers can be written as

decimals. There are two (and only 2) representations of Rational Numbers as decimals.

Representation 1: Termination.

If the decimal terminates, it is rational.

Ex: 2.313 is Rational

Ex: .1 is Rational

Ex: -.5536423 is Rational

Representation 2: Infinite Pattern.

Ex: -1.555... is rational. The ellipsis here say the pattern "555" continues forever.

Ex: 0.010101... is rational

Ex: 105.99199919... is rational.

IRRATIONAL NUMBERS

The irrational numbers are all numbers that are not Rational. Remember that the following are all irrational:

Ex. π is irrational

Ex: e is irrational

Ex: $\sqrt{2}$ is irrational

The negatives of each are also irrational.

Decimal representation: There is one decimal representation for irrational numbers. This is a non-terminating decimal without an infinitely-repeating pattern. Some examples:

Ex: $-2.312974\ldots$ is irrational.

The "..." ellipsis in this case tells you the decimal never ends, but you will notice no pattern is evident in the first 6 decimal places. Thus the decimal goes on without end and without a pattern.

Ex: $-1.23296\ldots$ is irrational

Ex: $0.1151934\ldots$ is irrational

Ex: $423.4343937\ldots$ is irrational

Notice that the ellipsis "..." are NOT mere eye candy. You can't ignore them.

Ex: $1.323157\ldots$ is irrational, but 1.323157 is rational since it terminates.

REAL NUMBERS

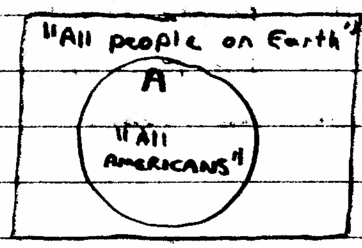
The Real numbers are all numbers that fall into the previous 5 categories. Every number we work with in this class will be a real number.

Definition: A Venn diagram uses a rectangle and one or more circles to represent relationships among sets.

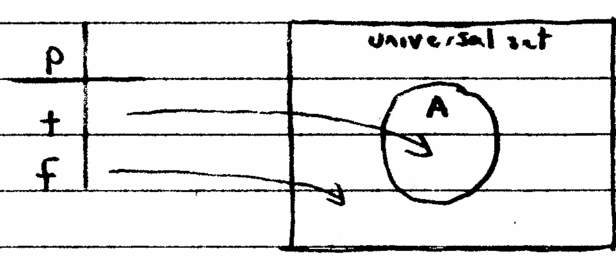
Consider the following, where P is a proposition:

P = "This person is an American"

We look at each person, ask if he/she is an American, and place the Americans in a set. Lets call this set "A", the set of Americans. But what people do we look at? Let's say we will look at all people on Earth. Thus "All people on Earth" will be the Universal Set... the set of every object we analyze with proposition P. Here is the Venn diagram:...

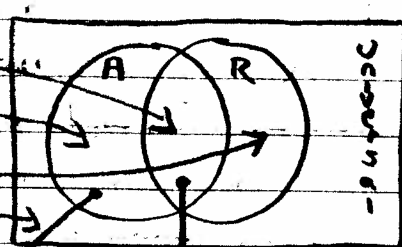


The circle contains only Americans on Earth. The outside of the rectangle includes everything we don't consider, such as dogs, planes, and people who are NOT on Earth. The area outside the circle but still inside the rectangle are all "other" people who do not classify as Americans. Using the truth table for P, we have:



Now, let q = "This person is a Russian."
Then we have the following, where R is the set of all Russians.

A	B
T	T
T	F
F	T
F	F

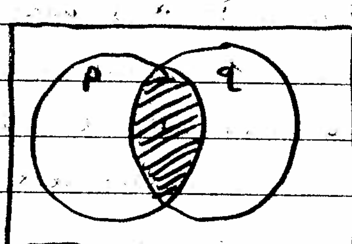


This region has all Americans who are NOT Russians.

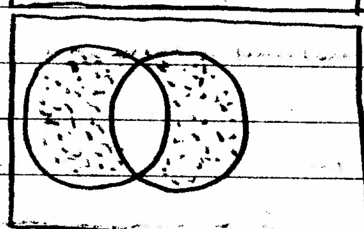
This region contains all people who are both Americans and Russians.

The other regions should be clear.

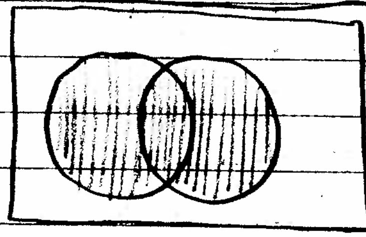
The logical operators for two sets are as follows:



= $p \wedge q$

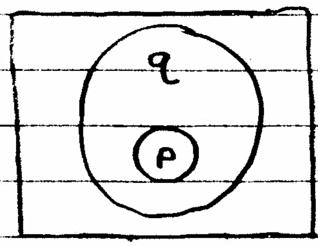


= p XOR q



= $p \vee q$

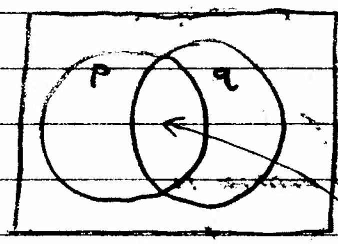
The three set relationships:



"Subset"

* p is a subset of q

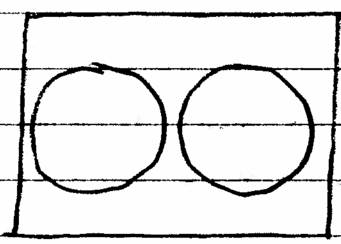
NOTE: you cannot reverse p and q here! Order matters.



Overlapping Sets

* p and q are overlapping sets

NOTE: This region contains members of both p and q



Disjoint Sets

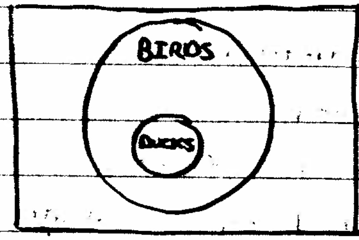
* p and q are disjoint sets.

IE: p and q have NO common members.

The four categorical propositions:

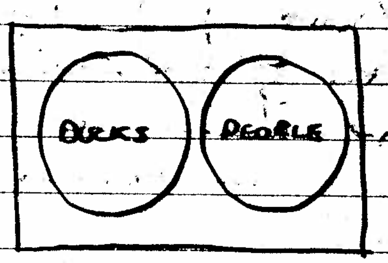
① All S are P. - This means all members of S are also members of P.
NOTE: This says "S is a subset of P."

Ex: All ducks are birds.
We write the venn diagram assuming "all animals" is the universal set:



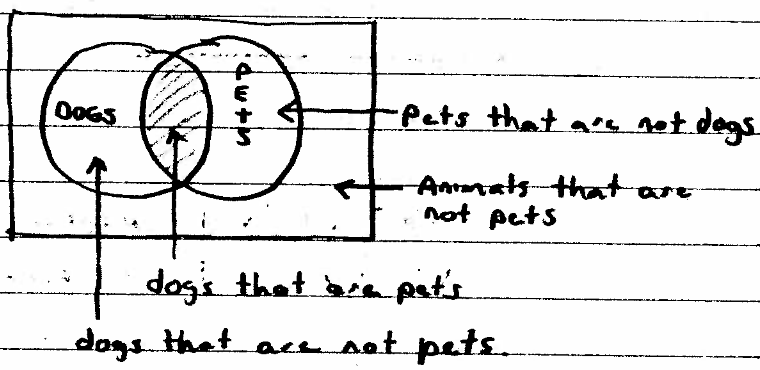
② No S are P - This means that S and P are disjoint sets:

Ex: No ducks are people



③ Some S are P - This says that at least one member of the set S is a member of set P.

Ex: Some dogs are pets

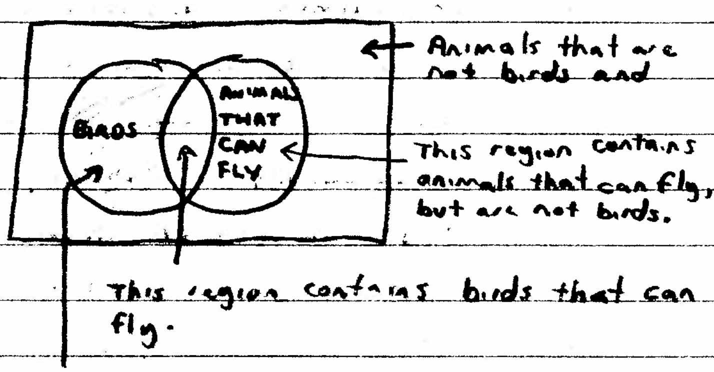


This claims the shaded region has at least 1 member. IE, it is NOT empty. However, the other regions, logically, might be empty.

(4) Some S are not P. - This says that there is at least 1 member of S that is not a member of set P.

Ex: Some birds cannot fly.

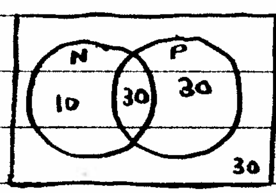
We rephrase this as: "Some birds are animals that cannot fly."



This region contains flightless birds. Note the logical argument "Some S are not P" says this region is NOT empty. The other regions could be empty.

Venn diagrams: some sample problem types:

Ex: Problem 74 on page 50

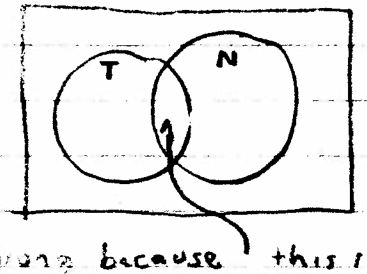


N = Nissan

P = Pickup

There are other answers. Just make sure the sets represented by the circles are NOT disjoint. If they are, you won't be able to place all the information on the diagram.

IE, this is WRONG:

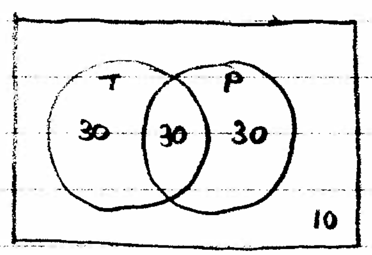


N = Nissan

P = Toyota

This is wrong because this region is empty. Why? Simple. Toyota never makes Nissan vehicles and Nissan never makes Toyotas. Likewise, selecting P = pickup and S = SUV is also a problem since P and S are disjoint. IE, no pickup is an SUV and no SUV can be a pickup.

Selecting T = Toyota and P = pickup is ok since these are not necessarily disjoint. Toyota makes pickups. If we chose these, our answer would be:



T = Toyota

P = Pickup

There are other possible answers.

Ex: Write this chart as a Venn Diagram:

Biology	Business	
32	110	Women
21	87	Men

This type of problem is easy. Select one row and one column, making sure they are not clearly disjoint. For instance, women and men are disjoint. Circle your chosen row and column.

Biology	Business	
32	110	Women
21	87	Men

Let these "circles" be the two circle "sets" in the venn diagram.